An Introduction to the Theory of Numbers

by

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Errata

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This list of errata records changes made since the version of March 1, 2004.

Version of July 31, 2011:

(1) On page 62, add the footnote:

Publisher's note: Pieter Moree discusses this theorem and proof in "A top hat for Moser's four mathematical rabbits," *The American Mathematical Monthly*, **118** (2011), 364–370.

Version of March 31, 2009:

(1) In "Unsolved Problems and Conjectures", page 85, number 42, add the footnote:

Publisher's note: The three examples he gives are the first of an infinite sequence of solutions—if the last term of one solution is $\frac{1}{M}$, replace it by $\frac{1}{M+1} + \frac{1}{M(M+1)}$ to get the next—and these solutions are well known, see, e.g., Wikipedia, Sylvester's sequence,

http://en.wikipedia.org/wiki/Sylvester%27s_sequence

(as of Mar. 31, 2009, 17:26 GMT). This is not the only family of solutions known today, see, e.g., Wikipedia, Znám's problem,

http://en.wikipedia.org/wiki/Zn%C3%A1m%27s_problem

(as of Mar. 31, 2009, 17:42 GMT). I don't know precisely what Moser meant by this conjecture.

Thanks to Joe Lipman for the references and helpful discussions.

Version of December 29, 2007:

(1) Chapter 5, Congruences. On page 51, at the end of the sentence

Nevertheless, in spite of numerous attempts this result of Vinogradov has not been much improved.

add the footnote

> **Publisher's note:** Very soon after these notes were written, the exponent of p in each of the two previous displayed formulas was effectively halved; see "The distribution of quadratic residues and non-residues," by D. A. Burgess, *Mathematika* (**4**), 1957, pp. 106–112.

Thanks to Harald Helfgott for this reference.

Version of October 1, 2007:

(1) Miscellaneous Problems, Problem 23. Change

$$n^{\varphi(n)} \prod \left(\frac{n!}{d^d} \right)^{\mu\left(\frac{n}{d}\right)}$$

to

$$n^{\varphi(n)} \prod_{d|n} \left(\frac{d!}{d^d} \right)^{\mu\left(\frac{n}{d}\right)}.$$

Thanks to John Williams for this correction.

Version of January 7, 2007:

(1) Page 2, change "$F_0 = 1$" to "$F_0 = 0$".
(2) Page 4, change

$$\frac{1}{(1-x)(1-x^2)(1-x^3)\dots} = 1 + x^1 + x^2 + x^3 + \cdots$$

to

$$\frac{1}{(1-x)(1-x^3)(1-x^5)(1-x^7)\dots} = (1+x)(1+x^2)(1+x^3)(1+x^4)\cdots.$$

(3) Page 11, change
$$\sigma_k(n) = \sum_{d|n} d^k \cdot \ell(n)$$

to
$$\sigma_k(n) = \sum_{d|n} (d^k \cdot 1).$$

Change "$\sigma = I_1 \times I_1$" to "$\sigma = I_0 \times I_1$".
(4) Page 13, change
$$\sum_{d|n} \Lambda(n) = \log n$$

to
$$\sum_{d\mid n} \Lambda(d) = \log n.$$

Change
$$\Lambda(d) = -\sum_{d\mid n} \mu(d)\log\left(\frac{n}{d}\right) = \sum_{d\mid n} \mu(d)\log d$$

to
$$\Lambda(n) = \sum_{d\mid n} \mu(d)\log\left(\frac{n}{d}\right) = -\sum_{d\mid n} \mu(d)\log d.$$

(5) Page 14, change
$$\sum \frac{\varphi(n)}{n^s} = \frac{1}{\zeta(s)}$$

to
$$\sum \frac{\mu(n)}{n^s} = \frac{1}{\zeta(s)}.$$

(6) Page 18, change "But now this n is fixed so there will also be an n such that ..."
to "But now this n is fixed so there will also be an m such that ..."

(7) Page 21, indicate that
$$\binom{2n}{n} > \frac{4^n}{2n}$$

only for $n > 1$.

(8) Page 27, change
$$F(n) = \sum_{d\mid n} f(d)$$

to
$$F(n) = \sum_{d=1}^{n} f(d).$$

(9) Page 28, top of page, fix tableau expression for $\mathcal{F}(x)$ and later add commentary to indicate that the second set of sums is taken over certain diagonal lines. (Unfortunately, this changed the page numbering in the rest of this chapter.)

(10) Page 38, change
$$0 < f(x)\sin x < \frac{n^n a^n}{n!} \to 0$$

to
$$0 < f(x)\sin x < \frac{\pi^n a^n}{n!} \to 0$$

(11) Page 43, change "Thus $2 \times 3 = 4 \times 3 \pmod 6$ does not imply $2 \equiv 3 \pmod 6$" to "Thus $2 \times 3 = 4 \times 3 \pmod 6$ does not imply $2 \equiv 4 \pmod 6$".

Thanks to Mark Hudson for these suggestions.

Version of October 4, 2004:

(1) On page 26, change "Every integer > 7 can be written as the sum of distinct primes" to "Every integer > 6 can be written as the sum of distinct primes."

(2) On page 73, change "Is every number > 2 the sum of two primes?" to "Is every even number > 2 the sum of two primes?"

Version of May 14, 2004:

(1) Add a reference to P. Erdös and J. L. Selfridge, "The products of consecutive integers is never a power," Illinois J. Math. **19** (1975), 292–301 in a footnote to Problem 40 in the list of Unsolved Problems and Conjectures. Thanks to Tony Noe for the reference.