

## Basic Concepts of Mathematics

by  
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*Errata*

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This list of errata records changes made since the version of October 29, 2001.

Version of May 19, 2017

- (1) Chapter 1, beginning of §1, change “The theory of sets, initiated by the German mathematician G. Cantor (1842–1918) ...” to “The theory of sets, initiated by Georg Cantor (1845–1918) ...”.
- (2) Chapter 1, §2, footnote 4, change “After the English logician John Venn (1834–1883)” to “After the logician John Venn (1834–1883)”.
- (3) Chapter 1, §2, footnote 6, change “Augustus de Morgan, Indian-born English mathematician and logician (1806–1871)” to “Augustus de Morgan, mathematician and logician (1806–1871).”.
- (4) Chapter 2, beginning of §2, change “(R. Dedekind, German mathematician, 1831–1916)” to “(Richard Dedekind, 1831–1916)”.
- (5) Chapter 2, §16, Example (a), change “ $x_n = 1/n$ ” to “ $x_n = 1/n, n = 1, 2, \dots$ ” and “Since all  $p_n$  are 0 so is  $\overline{L} = \sup_n p_n$ ” to “Since all  $p_n$  are 0 so is  $\underline{L} = \sup_n p_n$ ”.
- (6) Chapter 3, §8, Problem 5(g), change “ $(z + 1 + i)(z + i - i)(z - 1 + i)(z - 1 - i)$ ” to “ $(z + 1 + i)(z + 1 - i)(z - 1 + i)(z - 1 - i)$ ”.

Version of February 3, 2014

- (1) Chapter 2, §4, Problem 5(iv), change “What if  $x < 0$  or  $y < 0$ ?” to “What if  $x > 0 > y$  or  $0 > x > y$ ?”.

Version of April 11, 2012

- (1) Chapter 1, §4, Problem 16(iii), change
  - (iii) If  $A \times B = (A' \times B') \cup (A'' \times B'')$ , with all three products not void, then either  $A' = A'' = A$  and  $B = B' \cup B''$ , or  $B' = B'' = B$  and  $A = A' \cup A''$ .to
  - (iii) If  $A \times B = (A' \times B') \cup (A'' \times B'')$ , with all three products not void, then we have  $A = A' \cup A''$ ,  $B = B' \cup B''$ , and at least one of  $A' = A''$ ,  $B' = B''$ ,  $A' \times B' \subseteq A'' \times B''$ , or  $A'' \times B'' \subseteq A' \times B'$ .

Thanks to José Pedro Quintanilha for this correction.

Version of January 12, 2012

(1) Chapter 1, §4, Problem 11, change

11. Let  $R$  be a transitive relation on  $A$ , and let  $S = \{(x, y) \mid xRy, (y, x) \notin R\}$ . Show that  $S$  is transitive and trichotomic on  $A$ . Is it true that the relation  $T = \{(x, y) \mid xRy, yRx\}$  is reflexive, symmetric, and transitive on  $A$ ? Is it so on some subset  $B \subseteq A$ ?

to

11. Let  $R$  be a transitive relation on  $A \neq \emptyset$  with  $D_R = D'_R = A$ , and let  $S = \{(x, y) \in R \mid (y, x) \notin R\}$ . Prove that  $S$  is transitive and show by example that it may or may not be trichotomic.

Thanks to José Pedro Quintanilha for providing the examples.

Version of February 12, 2007

(1) Chapter 2, §14, proof of Theorem 2, change

$$r' = \frac{m'}{n'} \quad (m', n' \in N)$$

to

$$r' = \frac{m'}{n'} \quad (m', n' \text{ naturals in } F).$$

In the first paragraph after the proof, change “Thus, we have also proved that *the set  $F$  of all natural numbers ...*” to “Thus, we have also proved that *the set  $N$  of all natural numbers ...*”.

Thanks to Jonathan Ferron for his comments.

Version of January 5, 2007

(1) Chapter 2, §14, Problem 6, change

$$(\forall n \in N) \quad (n) = n \cdot 1',$$

to

$$(\forall n \in N) \quad f(n) = n \cdot 1'.$$

(2) Chapter 2, §15, Example 2, change

$$A = \{x \in R \mid x \leq 0 \text{ or } x^2 \leq 2\}, \quad B = \{x \in R \mid x > 0, x^2 > 2\}$$

to

$$A = \{x \in R \mid x \leq 0 \text{ or } x^2 < 2\}, \quad B = \{x \in R \mid x > 0 \text{ and } x^2 > 2\}.$$

(3) Chapter 2, §16, after Corollary 1, change “Also,  $\bar{L} \geq p_n \dots$ ” to “Also,  $\underline{L} \geq p_n \dots$ ”.

(4) Chapter 2, §16, Proof of Theorem 1, part (i), change “such  $n$  must occur in *each* set  $A_n = \{x_m, x_{m+1}, \dots\}$ ” to “such  $x_n$  must occur in *each* set  $A_m = \{x_m, x_{m+1}, \dots\}$ ”.

(5) Chapter 3, §1, Problem 1, change “ $2\vec{u} - \vec{v} - 3\vec{w} + 5\vec{w}$ ” to “ $2\vec{u} - \vec{v} - 3\vec{w} + 5\vec{x}$ ”.

(6) Chapter 3, §1, Problem 4, change “express  $\vec{w}$  as a linear combination of  $\vec{u}, \vec{v}, \vec{w}$ ,  $\vec{x}$ ” to “express  $\vec{x}$  as a linear combination of  $\vec{u}, \vec{v}, \vec{w}$ ”.

Thanks to Gregory Hersh for his comments.

Version of October 4, 2006:

- (1) Chapter 2, §7, “Induction Law for Integers,” change “ $p(n)$ ” to “ $P(n)$ ”.
- (2) Chapter 2, §13, Problem 2, change

$$\sum_{k=1}^r cr^k \text{ to } \sum_{k=0}^n cr^k \text{ (twice).}$$

- (3) Chapter 2, §14, Definition 2 and just before: We clarify that  $n \cdot a$  and  $r \cdot a$  mean the same as  $na$  and  $ra$ , respectively. Later on same page, change “Moreover, if  $r \in N$ , i.e.,  $r = m/l \dots$ ” to “Moreover, if  $r \in N$ , i.e.,  $r = m/1 \dots$ ”.
- (4) Chapter 3, §5, Problem 5, change  $kP + kk'P' = 0$  to  $kP + k'P' = 0$ .  
Thanks to Gregory Hersh for his comments.

Version of August 14, 2006:

- (1) Last line of Definition 3 of Chapter 2, §6, change

$$\prod_{k=1}^n x_k = x_1 \text{ to } \prod_{k=1}^1 x_k = x_1.$$

Thanks to Gregory Hersh for this change.

Version of March 16, 2005:

- (1) Chapter 2, §9, Theorem 1, change “nfimum” to “infimum”.
- (2) After Chapter 3, §5, Theorem 1, change “uit” to “unit”.
- (3) Chapter 3, §9, part II, change definition of complex dot product

$$x \cdot y = \sum_{k=1}^n x_k \bar{y}_k \text{ to } x \cdot y = \sum_{k=1}^n x_k \bar{y}_k.$$

Thanks to “Plareplane” for these three changes.

Version of September 25, 2004:

- (1) In Chapter 1, §5, Definition 1, change “iff the image  $R[x]$  or every element  $x \in D_R$ ” to “iff the image  $R[x]$  for every element  $x \in D_R$ .”

Version of August 2, 2004:

- (1) Remove the “(in preparation)” note for *Mathematical Analysis I*.

Version of March 11, 2004:

- (1) Add entries in the Table of Contents for the Preface and the section “About the Author”.

Version of March 1, 2004:

- (1) Chapter 1, §2, Theorem 2: Change

$$(i) \ E - \bigcup_i A_i = \bigcap_i (E - A_j)$$

to

$$(i) \quad E - \bigcup_i A_i = \bigcap_i (E - A_i).$$

- (2) Reorder frontmatter to publishing standards; recenter displayed equations in problem sets; ensure that new chapters start on odd-numbered pages. Unfortunately, this added five (blank) pages to the text and changed page numbers.

Version of December 14, 2001:

- (1) In the first line of Chapter 2, §16, Definition 1, change  $\{x_n \subseteq E^*\}$  to  $\{x_n\} \subseteq E^*$ .